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Electromagnetic Effects in Unified Field Theory

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In unified field theory we derive expressions for the electric current densities j and ρ. We show that j and ρ depend on the intensities **E** and **H**; E and H possess a common limit $1/\alpha$; and Coulomb's law is not compatible with the unified theory.

1. PREPARATIONS

A. Einstein and B. Kaufman's work (1954) on the eigenvalues of gi_k is our starting point. They introduce a system of reference connected with the "diagonal system" of g^i_k (both are inertial systems) by a Lorentz transformation and give a matrix for g_{ik} in their system. E–K's matrix is unfilled, which is inconvenient for our work.

In preparation, we first transform E–K's matrix to a filled matrix of g_{ik} by a general Lorentz transformation. Next we find the contravariant components g^{ik} of our matrix, and connect g_{ik} with the intensities **E** and **H**.

In the unified theory and in the inertial system moving relatively to E–K's system with uniform velocity, we give a filled matrix for g_{ik} as

$$(g_{ik})=\begin{pmatrix} 1 & \underset{\vee}{g_{12}} & \underset{\vee}{g_{13}} & \underset{\vee}{g_{14}} \\ -\underset{\vee}{g_{12}} & 1 & \underset{\vee}{g_{23}} & \underset{\vee}{g_{24}} \\ -\underset{\vee}{g_{13}} & -\underset{\vee}{g_{23}} & 1 & \underset{\vee}{g_{34}} \\ -\underset{\vee}{g_{14}} & -\underset{\vee}{g_{24}} & -\underset{\vee}{g_{34}} & -1 \end{pmatrix} \equiv \begin{pmatrix} 1 & U_z & -U_y & -V_x \\ -U_z & 1 & U_x & -V_y \\ U_y & -U_x & 1 & -V_z \\ V_x & V_y & V_z & -1 \end{pmatrix} \tag{1}$$

and

$$\mathbf{U}=U_x\mathbf{i}+U_y\mathbf{j}+U_z\mathbf{k}, \qquad \mathbf{V}=V_x\mathbf{i}+V_y\mathbf{j}+V_z\mathbf{k},$$

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which is derived from E–K's matrix

$$(g'_{ik})=\begin{pmatrix} 1 & g'_{\underset{\vee}{12}} & & \\ -g'_{\underset{\vee}{12}} & 1 & & 0 \\ & & 1 & g'_{\underset{\vee}{34}} \\ 0 & & -g'_{\underset{\vee}{34}} & -1 \end{pmatrix}$$

by a general Lorentz transformation.

Next, from (1), using $\mathbf{U}$ and $\mathbf{V}$ for brevity, we derive the contravariant components g^{ik} as

$$\begin{aligned}
g^{11}&=\frac{1}{g}\left[-1+\mathbf{V}^2-\left(U_x^2+V_x^2\right)\right], \quad g^{22}=\frac{1}{g}\left[-1+\mathbf{V}^2-\left(U_y^2+V_y^2\right)\right]\\
g^{33}&=\frac{1}{g}\left[-1+\mathbf{V}^2-\left(U_z^2+V_z^2\right)\right], \quad g^{44}=\frac{1}{g}\left[1+\mathbf{U}^2\right]\\
g^{12}, g^{21}&=\frac{1}{g}\left[\mp U_z \pm (\mathbf{U}\cdot\mathbf{V})V_z-\left(U_xU_y+V_xV_y\right)\right]\\
g^{31}, g^{13}&=\frac{1}{g}\left[\mp U_y \pm (\mathbf{U}\cdot\mathbf{V})V_y-\left(U_zU_x+V_zV_x\right)\right]\\
g^{23}, g^{32}&=\frac{1}{g}\left[\mp U_x \pm (\mathbf{U}\cdot\mathbf{V})V_x-\left(U_yU_z+V_yV_z\right)\right]\\
g^{14}, g^{41}&=\frac{1}{g}\left[\mp V_x \mp (\mathbf{U}\cdot\mathbf{V})U_x+\left(U_yV_z-U_zV_y\right)\right]\\
g^{24}, g^{42}&=\frac{1}{g}\left[\mp V_y \mp (\mathbf{U}\cdot\mathbf{V})U_y+\left(U_zV_x-U_xV_z\right)\right]\\
g^{34}, g^{43}&=\frac{1}{g}\left[\mp V_z \mp (\mathbf{U}\cdot\mathbf{V})U_z+\left(U_xV_y-U_yV_x\right)\right]
\end{aligned} \tag{2}$$

where

$$g=\det(g_{ik})=-\left[1+(\mathbf{U}^2-\mathbf{V}^2)-(\mathbf{U}\cdot\mathbf{V})^2\right],$$

Einstein defines the tensor density $\mathfrak{g}_{ik}=-g^{1/2}g_{ik}$ and suggests $\mathfrak{g}^{\underset{\vee}{ik}}$ is the intensity Einstein (1955). From this and (2), we get

$$\begin{aligned}
\alpha\mathbf{E}&=\mathfrak{g}^{\underset{\vee}{23}}\mathbf{i}+\mathfrak{g}^{\underset{\vee}{31}}\mathbf{j}+\mathfrak{g}^{\underset{\vee}{12}}\mathbf{k}=\frac{1}{(-g)^{1/2}}\left[\mathbf{U}-(\mathbf{U}\cdot\mathbf{V})\mathbf{V}\right]\\
\alpha\mathbf{H}&=\mathfrak{g}^{\underset{\vee}{14}}\mathbf{i}+\mathfrak{g}^{\underset{\vee}{24}}\mathbf{j}+\mathfrak{g}^{\underset{\vee}{34}}\mathbf{k}=\frac{1}{(-g)^{1/2}}\left[\mathbf{V}+(\mathbf{U}\cdot\mathbf{V})\mathbf{U}\right]
\end{aligned} \tag{3}$$

where α is the inverse of the maximum limit of E and H as clarified later.

From (3), we have the expression of $g_{\underset{\vee}{ik}}$ in terms of **E** and **H**

$$\mathbf{U}=g_{\underset{\vee}{23}}\mathbf{i}+g_{\underset{\vee}{31}}\mathbf{j}+g_{\underset{\vee}{12}}\mathbf{k}=\alpha\frac{\mathbf{E}+\alpha^2(\mathbf{E}\cdot\mathbf{H})\mathbf{H}}{h^{1/2}}$$

$$\mathbf{V}=g_{\underset{\vee}{41}}\mathbf{i}+g_{\underset{\vee}{42}}\mathbf{j}+g_{\underset{\vee}{43}}\mathbf{k}=\alpha\frac{\mathbf{H}-\alpha^2(\mathbf{E}\cdot\mathbf{H})\mathbf{E}}{h^{1/2}} \tag{4}$$

where $h=1-\alpha^2(\mathbf{E}^2-\mathbf{H}^2)-\alpha^4(\mathbf{E}\cdot\mathbf{H})^2$.

2. CONCLUSIONS

Now we derive the expressions of the current densities in terms of **E** and **H** by (4). Einstein (1955) derives the electromagnetic equations as

$$\mathfrak{g}^{is}_{,s}=0 \tag{5}$$

$$a^m=\frac{1}{6}\eta^{iklm}(g_{\underset{\vee}{ik},l}+g_{\underset{\vee}{kl},i}+g_{\underset{\vee}{li},k}) \tag{6}$$

where a^m is the specific current density. In (5) and (6) the field variables are different, which is a remarkable property. It is this property that causes some electromagnetic effects.

Next, we rewrite (5) and (6) in their usual forms:

$$\operatorname{rot}\mathbf{E}+\frac{\partial\mathbf{H}}{C\partial t}=0,\ \operatorname{div}\mathbf{H}=0; \tag{5a}$$

and

$$\alpha\mathbf{j}=\operatorname{rot}\mathbf{V}-\frac{\partial\mathbf{U}}{C\partial t},\ \alpha\rho=\operatorname{div}\mathbf{U}, \tag{6a}$$

where **j** and ρ are the Einstein current densities. Before this there held

$$\mathbf{j}_0=\operatorname{rot}\mathbf{H}-\frac{\partial\mathbf{E}}{C\partial t}\quad\text{and}\quad\rho_0=\operatorname{div}\mathbf{E} \tag{7}$$

where $\mathbf{j}_0$ and ρ_0 are the Maxwell current densities.

Last, calculating (6a) with (4), (5a) and (7), we obtain

$$\mathbf{j}=\frac{1}{h^{1/2}}j_0+\frac{\alpha^2}{h^{1/2}}\left[\mathbf{E}\times\nabla(\mathbf{E}\cdot\mathbf{H})-\mathbf{H}\frac{\partial}{C\partial t}(\mathbf{E}\cdot\mathbf{H})\right]$$

$$-\left[\mathbf{H}-\alpha^2(\mathbf{E}\cdot\mathbf{H})\mathbf{E}\right]\times\nabla\left(\frac{1}{h^{1/2}}\right)-\left[\mathbf{E}+\alpha^2(\mathbf{E}\cdot\mathbf{H})\mathbf{H}\right]\frac{\partial}{C\partial t}\left(\frac{1}{h^{1/2}}\right) \quad (8)$$

$$\rho=\frac{1}{h^{1/2}}\rho_0+\frac{\alpha^2}{h^{1/2}}\mathbf{H}\cdot\nabla(\mathbf{E}\cdot\mathbf{H})+\left[\mathbf{E}+\alpha^2(\mathbf{E}\cdot\mathbf{H})\mathbf{H}\right]\cdot\nabla\left(\frac{1}{h^{1/2}}\right) \quad (9)$$

The physical meaning of (8) and (9) can be interpreted as below. $\mathbf{j}$ and ρ are divided into two parts. The leading terms in (8) and (9) say that Maxwell's currents $\mathbf{j}_0$ and ρ_0 are exchanged for Einstein's currents in the ratio $1/h^{1/2}$. The remaining terms are additional currents induced by the variation of **E** and **H** in space-time in company with **E** and **H** themselves, besides Maxwell's currents.

When $\mathbf{E},\mathbf{H}\to 0$, $\mathbf{j}\to\mathbf{j}_0$, $\rho\to\rho_0$, then $\mathbf{j}_0, \rho_0$, and (7) become the corresponding things in a weak electromagnetic field.

These expressions hold for any type of electromagnetic field in any inertial system, since they are derived from Einstein and Kaufman's work without any hypothesis.

From (8) and (9) we come to the following conclusions.

(1) $\mathbf{j}$ and ρ depend on E and H.

Above all, if the additional currents with $O(\alpha^2)$ can be omitted, and if $\mathbf{j}=\rho\mathbf{V}$, then (8) and (9) reduce uniquely to

$$\rho=\rho_0/\left[1-\alpha^2(\mathbf{E}^2-\mathbf{H}^2)-\alpha^2(\mathbf{E}\cdot\mathbf{H})^2\right]^{1/2}$$

and the density depends on E and H rather clearly.

(2) The intensities possess a common upper limit, that is,

$$|E|,|H|<1/\alpha$$

By putting $H=0$ into the factor $1/h^{1/2}$, the limit to E is obtained. (To expose the nature of E, we should let it act alone.) The limit to H arises not from $1/h^{1/2}$ again, but from the Lorentz transform

$$E'=\left(E\pm\frac{V}{C}H\right)/(1-V^2/C^2)^{1/2}$$

From this, if E and E' possess the limit $1/\alpha$, H must also.

The physical meaning of α can now be clarified. It is the inverse of the limit to E and H.

(3) Coulomb's law is incompatible with the unified theory.

The Coulomb law $\mathbf{E}=q\mathbf{r}/r^3$, $\mathbf{H}=0$, with $h=1-\alpha^2q^2/r^4$ naturally satisfies (5a) and (7), and fits with (8). If it is put into (9), we get the density of additional electric charge as

$$\rho=\mathbf{E}\cdot\nabla\left(\frac{1}{h^{1/2}}\right)=-\frac{2\alpha^2q^3}{r\left[\left(r^4-\alpha^2q^2\right)^3\right]^{1/2}}$$

where when $r\leqslant(\alpha q)^{1/2}$, ρ is meaningless. That ρ differs from

$$\rho_0=\operatorname{div}\mathbf{E}=0$$

is understandable, but the additional charge

$$Q=\int_{\sqrt{\alpha q}+\epsilon}^{\infty}\rho\cdot 4\pi r^2\,dr=\left.\frac{4\pi q r^2}{\left(r^4-\alpha^2q^2\right)^{1/2}}\right|_{\sqrt{\alpha q}+\epsilon}^{\infty}\to-\infty$$

is mysterious.[1] Together with the consideration that E itself has no limit, this leads us to say that Coulomb's law is not compatible with the unified theory.

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[1]Even if we introduce the image density of electric charge, the divergence of Q is unavoidable, unless we disregard the singular point $r=(\alpha q)^{1/2}$ of ρ, and extend the integral as from 0 to ∞. If we do so, then $Q=4\pi q$, while the mathematical conception of the singular point would be more ambiguous.